# Number of Women in Physics Departments: A Simulation Analysis 

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Women's representation in physics lags behind most other STEM disciplines. Currently, women make up about $13 \%$ of faculty members in all physics degree-granting departments, and there are physics departments with no women faculty members at all. These two data points are often cited as evidence of a lack of equity for women.

The number of departments with no women among their faculty members seems appealing as a measure of equity by sex. At first glance, it may seem that as the representation of women in physics increases, all departments "should" have at least one woman faculty member. However, we will show that this is an unlikely outcome. In fact, if we took all of the women in bachelor's-granting departments and assigned each one to a different department, almost one-third of the departments would still have no women among their faculty members.

In this article, we demonstrate that two factors affect the representation of women in a single physics department:

- the number of faculty members in the single department and
- the overall proportion of women among all current physics faculty. There are a number of physics departments with only two or three faculty members; it is unlikely that these departments will have a woman among the faculty because the overall representation of women among all physics faculty members is low. We will examine this point more closely using the binomial distribution which includes two parameters: $n$ and $p$. In this analysis, the number of faculty members in a single physics department is $n$, and the overall proportion of women among all current physics faculty members is $p$. Our results suggest that that we should not accept the absence of women in a particular department as evidence that the department is biased against women.

We begin by presenting the current situation of women's representation in physics departments. Next, we present the results of simulations that

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calculate the expected number of departments that have no women faculty members. These simulations are based on actual data about the profile of degree-granting physics departments and the representation of women in physics. Using these data and applying the binomial distribution, we calculate the expected proportions of departments with no women.

Finally, we compare these expected proportions to the actual proportions of departments with no women. We are able to show that the actual proportion of departments with no women is lower than expected for PhDgranting departments. For bachelor's-granting departments, the actual proportion of departments with no women is about the same as the expected proportion.

We also include an analysis using hypothetical values for the proportion of women among the physics faculty to examine what might happen as the representation of women increases.

## The Current Situation

Women make up $13 \%$ of professorial-rank faculty members (assistant, associate, and full professors) overall. However, the proportion of women is different in PhD-granting physics departments than it is in bachelor'sgranting physics departments. While women's representation is higher in bachelor's-granting departments ( $16 \%$ versus $11 \%$ ), the bachelor's departments average fewer faculty members than departments which grant doctorates (Table 1).

Women's representation in physics has been slowly increasing. Yet there remain many departments that have no women on the faculty. In fact, almost half of bachelor's departments and $8 \%$ of PhD departments have faculties that are all the same sex. (Most of these single-sex departments have no women; there are a few with no men.) This paper will explain that much of the reason for this has to do with the number of physics faculty members in each department and the representation of women among physics faculty members overall.

Table 1
Basic Data on Faculty Members in Degree-Granting Physics Departments

| Highest Physics Degree Awarded | Bachelor's | PhD |
| ---: | :---: | :---: |
| Smallest Department (\# of faculty members) | 1 | 3 |
| Median Size (\# of faculty members) | 4 | 22 |
| Largest Department (\# of faculty members) | 27 | 75 |
| Women's Representation among <br> Professorial-Rank Faculty Members | $16 \%$ | $11 \%$ |
| Departments That Have No Women | $47 \%$ | $8 \%$ |
| Departments That have No Men | $1 \%$ | $0 \%$ |
| Number of Departments (2010) | 503 | 192 |

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## Simulations of the Effects of Department Size and Proportion of Women

In order to illustrate the effects of the number of faculty members and the overall proportion of women on the number of departments with no women, we present the results of simulations. The simulated outcomes in this paper are calculated using the binomial distribution. As noted earlier, the binomial distribution has two parameters: $n$ and $p$. We use the binomial distribution to determine the number of women selected into a department consisting of a given total number of faculty members. The two parameters are $n$ and $p$ where
$n$ is the total number of faculty members in a given department and $p$ is the overall proportion of women among all current physics faculty (or, the probability that a randomly selected faculty member is a woman).
The probability of getting exactly $k$ women among $n$ faculty members is given by

$$
P(K=k)=\frac{n!}{k!(n-k)!} p^{k}(1-p)^{n-k}
$$

In Table 2, we used the binomial distribution to calculate the probability that departments have same-sex faculties, given a certain number of faculty members in a department and given a certain percentage of women faculty members in the system. We picked the number of faculty members $(n)$ to vary among $4,6,15$, and 22 . In this example, four- and six-member departments illustrate typical bachelor's-granting departments, while departments with 15 and 22 faculty members are illustrative of PhDgranting departments. We varied the proportion of women among all current faculty (p) from $11 \%$ to $50 \%$. We use $50 \%$ here to illustrate the situation if half of all faculty members were women.
For physics departments that award a bachelor's degree only:

- $16 \%$ is the actual proportion of women among the faculty and
- $23 \%$ is the actual proportion of women among assistant professors.

PhD-granting physics departments typically have more faculty members than bachelor's-granting departments. The size of the department is one factor that affects whether or not a department will have a woman among its faculty members.

For physics departments that award a doctorate:

- $11 \%$ is the actual proportion of women among the faculty and
- $20 \%$ is the actual proportion of women among assistant professors.

Physics departments that award only a bachelor's degree tend to have fewer faculty members.
Note that as the number of faculty members in a given department increases, the total probability that faculty members will be all the same sex decreases. Similarly, as the proportion of women overall increases, the total probability that a department has all faculty members of one sex decreases.

Table 2
Probability That Faculty is All Women or All Men
For Specific Department Sizes and \% Women Overall

| Total Number of Faculty Members in a Single Department (n) | Overall \% <br> Women Among <br> All Faculty <br> Members <br> (p) | Probability That Department Is ... |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | (A) <br> All Women $\begin{gathered} (\%) \\ (\mathrm{P}(k=n)) \end{gathered}$ | (B) <br> All Men $\begin{gathered} (\%) \\ (P(k=0)) \end{gathered}$ | One Sex $\begin{gathered} \text { (\%) } \\ (A+B) \end{gathered}$ |
| 4 | 16 | 0.1 | 50 | 50 |
| 4 | 23 | 0.3 | 35 | 35 |
| 4 | 50 | 6.25 | 6.25 | 12.5 |
| 6 | 16 | < 0.1 | 35 | 35 |
| 6 | 23 | < 0.1 | 21 | 21 |
| 6 | 50 | 1.6 | 1.6 | 3.1 |
| 15 | 11 | < 0.1 | 17 | 17 |
| 15 | 20 | < 0.1 | 4 | 4 |
| 15 | 50 | < 0.1 | << 0.1 | << 0.1 |
| 22 | 11 | < 0.1 | 8 | 8 |
| 22 | 20 | < 0.1 | 1 | 1 |
| 22 | 50 | < 0.1 | << 0.1 | < 0.1 |

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The effects of the number of faculty members on the probability that a department will have faculty members who are all the same sex helps explain why such a large percentage of bachelor's departments have no women. Bachelor's departments tend to be small; therefore, as we have seen in Table 2, there is a higher probability of these departments having no women.

To further illustrate the effects of department size, we use another example. We keep the proportion of women constant while varying only the size of the department. For illustration, consider keeping the proportion of women constant at $50 \%$ while varying only the size of the department. We simulate the random assignment of faculty members to departments of all one size. We repeated the simulation 500 times for each department size. The mechanics of the simulation are detailed in Figure 1, and the results are shown in Figure 2 (on page 6).

Figure 1
The Simulation Process


In our first
simulation, $n$ is constant. In our remaining simulations, $n$ changes to reflect the actual profile of all physics departments.

As the results show (Figure 2), it is harder to achieve diversity in smaller departments than in larger departments. When we randomly assign 1000 faculty members - half men and half women - into 500 departments with two faculty members in each department, about half of the departments have no variation by sex. This is not surprising; if we toss a fair coin twice, we will get one head and one tail about half of the time. Of course, as we increase the size of the department, the results are increasingly likely to show faculty members of both sexes. In this example with half men and half women overall, only when we reach six faculty members or more is there no variation by sex in less than $10 \%$ of the departments. (With five faculty members we are under $10 \%$ most, but not all, of the time.)

Figure 2

Even if half of all faculty members are women, small departments are likely to have faculty members that are all the same sex.

## We can calculate

 the expected proportion of physics departments that have faculty members of only one sex using the binomial distribution.

The box outlines the middle $50 \%$ of the results from 500 simulations with the bar indicating the median. Lines extend to the maximum and minimum values.
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## Expected and Actual Proportions of Same-Sex Departments

We can continue to examine the effects of department size on the sex composition of physics departments by looking at data from the Academic Workforce Survey of physics departments, which AIP conducts every two years. In Tables 3 and 4, we show the actual distribution of the number of faculty members in each department, along with the probabilities of departments with same-sex faculties (calculated using the binomial distribution).
In bachelor's-granting physics departments, the median number of professorial-rank faculty is four, and $16 \%$ of all professorial-rank faculty members are women. As shown in Table 3, combining the expected probability of departments that are same sex (Column C) with the actual proportion of departments of each size (Column B) yields an expected total of $49 \%$ of bachelor's-granting departments that will have same-sex faculties (total Column D). From the 2010 Academic Workforce Survey, we know that $48 \%$ of bachelor's-granting physics departments have no variation by sex ( $47 \%$ of these have no women and $1 \%$ have no men).
In PhD-granting physics departments, the median number of professorialrank faculty is 22 , and $11 \%$ of all professorial-rank faculty are women. As shown in Table 4, combining the data yields an expected total of $12 \%$ of PhD-granting departments that will have same-sex faculties. Currently, $8 \%$
of PhD departments are same sex (all of these have no women). PhDgranting departments typically have more faculty than bachelor's-granting departments, so they are more likely to have both sexes represented among faculty members. Therefore, the expected percentage of same-sex departments is about the same as or higher than the actual percentage in both bachelor's- and PhD-granting departments.

## Table 3

Expected Proportion of Departments With Same-Sex Faculties Bachelor's-Granting Physics Departments Only, 2009-10 Academic Year

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| Number of Professorial Rank Faculty Members | Proportion of Bachelor'sGranting Departments (\%) | Probability Department Has All One Sex* (\%) | Expected Proportion of Same-Sex Departments (Column B * Column C) (\%) |
| 1 | 5 | 100 | 5 |
| 2 | 15 | 73 | 11 |
| 3 | 17 | 60 | 10 |
| 4 | 17 | 50 | 9 |
| 5 | 16 | 42 | 7 |
| 6 | 9 | 35 | 3 |
| 7 | 8 | 30 | 2 |
| 8 | 8 | 25 | 1 |
| 9 | 3 | 21 | 1 |
| 10 | 2 | 17 | 0 |
| 11 | 1 | 15 | 0 |
| 12 | 1 | 12 | 0 |
| 13 | 1 | 10 | 0 |
| 14 | 0.2 | 9 | 0 |
| 18 | 0.2 | 7 | 0 |
| 20 | 0.2 | 6 | 0 |
| 21 | 0.2 | 5 | 0 |
| 27 | 0.2 | 4 | 0 |
| TOTAL | 100\% | - | 49\% |

* The probabilities in Column C assume that $16 \%$ of the faculty members are women.

The values in Columns B and D may not sum to total shown due to rounding. http://www.aip.org/statistics

Next, we simulated the random assignment of a fixed number of men and women faculty into departments of varying sizes to take a closer look at how expected proportions of same-sex faculties match reality.

The simulations rely on the binomial distribution, which we have described previously. We simulate the assignment of men and women in professorial ranks to each department, varying $n$ to account for the size of the current

Given the large number of small departments and the low representation of women among faculty members, we expect almost half of the bachelor's-granting physics departments to have same-sex faculties.
department and $p$ to account for the probability that a randomly selected faculty member is a woman. For the first bachelor's-granting department, $p$ $=16 \%$. For subsequent departments, the value of $p$ changes dynamically to account for the number of men and women still remaining to be assigned.

Table 4
Expected Proportion of Departments With Same-Sex Faculties PhD-Granting Physics Departments Only, 2009-10 Academic Year

Even though the representation of women is lower in PhD-granting physics departments, the expected proportion of departments with same-sex faculties is lower than for bachelor's-granting departments because PhD departments have more faculty members.

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| Number of <br> Professorial <br> Rank Faculty <br> Members | Proportion of <br> PhD-Granting <br> Departments <br> $(\%)$ | Probability <br> Department <br> Has All One <br> Sex* <br> $(\%)$ | Expected Proportion <br> of Departments with <br> All One Sex <br> (Column B |
| 3 | 1 | 71 | $(\%)$ |
| 4 | 1 | 63 | 0.4 |
| 5 | 2 | 56 | 0.3 |
| 7 | 1 | 44 | 0.9 |
| 8 | 2 | 39 | 0.2 |
| 9 | 1 | 35 | 0.9 |
| 10 | 2 | 31 | 0.2 |
| 11 | 3 | 28 | 0.7 |
| 12 | 4 | 25 | 0.9 |
| 13 | 4 | 22 | 1.1 |
| 14 | 4 | 20 | 0.8 |
| 15 | 2 | 17 | 0.9 |
| 16 | 5 | 15 | 0.4 |
| 17 | 4 | 14 | 0.8 |
| 18 | 4 | 12 | 0.6 |
| 19 | 3 | 11 | 0.5 |
| 20 | 3 | 10 | 0.3 |
| 22 or more | 53 | Varies | 0.3 |
| TOTAL | $\mathbf{1 0 0 \%}$ | - | 2 |

* The probabilities in Column C assume that $11 \%$ of the faculty members are women.
The values in Columns B and D may not sum to total shown due to rounding. http://www.aip.org/statistics

For example, assume there are initially 1000 faculty members to be assigned to various departments, 160 women and 840 men. Then $p$, the probability that a randomly selected faculty member is a woman, is 160/(840+160) = $16 \%$. However, $p$ will change for the next department based on how many men and women are assigned to the current department. If there were two women and four men assigned to the first department, then $p$ for the second department is $158 /(836+158)=15.9 \%$. The fact that the first
department had two women assigned to it reduces the probability that a randomly selected faculty member in the second department is a woman.

Figures 3 and 4 (following pages) depict the results of 500 simulations of the random assignment of men and women faculty into professorial ranks at bachelor's- and PhD-granting physics departments. Recall that the actual percentage of bachelor's-granting departments that have no women is $47 \%$. In over $74 \%$ of the 500 simulations, the percentage of bachelor's departments that had no women was higher than $47 \%$.

These results suggest that bachelor's departments are actually placing women faculty members into more departments than if the distribution of women into departments were left to chance. Of course, this also means that, given the set number of women, there are more departments with only one woman than there would be if the distribution was purely random. Furthermore, our results suggest that the lack of women in bachelor'sgranting departments is consistent with two facts:

- $16 \%$ of the faculty members in bachelor's departments are women, and
- bachelor's departments tend to have a small number of faculty members (median=4).

Again, recall that 8\% of PhD-granting physics departments have no women. In 500 simulations of the assignment of professorial rank faculty members to PhD-granting departments, $99 \%$ of the time, more than $8 \%$ of the departments had no women (Figure 3). In other words, women faculty members actually are found in more PhD departments than one would expect. This indicates that PhD-granting departments overall are succeeding in getting women faculty members into a higher number of departments than we would expect based on the size of the departments and the percentage of women in the system.

We know that faculty members are not randomly assigned to departments. There are many factors that affect the makeup of a physics department, including factors the department can impact and factors beyond the control of the department. The department considers the ability of its infrastructure and setting to support an applicant's research, the fit of the applicant's research with the department's mission and existing faculty members' areas of expertise, the ability of the applicant to secure external funding, and a variety of other factors including personalities. The applicant also has many things to consider, including externalities such as family issues and competing offers - both within and beyond academia.

In our simulations, the expected proportion of departments with same-sex faculties was higher than the actual 74\% of the time for bachelor's-granting physics departments and $99 \%$ of the time for physics departments which grant PhDs.

Figure 3

In 370 of our 500 simulation runs, there were more bachelor's-granting physics departments with same-sex faculties than there are in reality.

In almost all of our
500 simulation runs, there were more PhD-granting physics departments with same-sex faculties than there are in reality.

Simulation Results: Proportion of Departments With No Women in the Professorial Ranks Bachelor's-Granting Physics Departments


The gold bar indicates the simulation results that mirror the actual proportion of departments with no women in the professorial ranks.
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Figure 4
Simulation Results: Proportion of Departments With No Women in the Professorial Ranks PhD-Granting Physics Departments


The gold bar indicates the simulation results that mirror the actual proportion of departments with no women in the professorial ranks.

[^0]Our results suggest that there is no bias against hiring women in the system as a whole. This does not mean that selected individual departments do not discriminate or have environments that are less than welcoming to women. However, the lack of women in physics departments cannot be used as a measure of gender equity because the number of departments that have women is the result of:

- the number of faculty members in the single department and
- the overall proportion of women among all physics faculty.


## What if Half of All Physics Faculty Members Were Women?

Even if half of all faculty members were women, we would still expect to find no gender variation in $20 \%$ of the bachelor's-granting physics departments - with $10 \%$ having no men and $10 \%$ having no women. We would expect almost all of the PhD-granting departments to include at least one woman and at least one man among the faculty (Table 5).

The percentage of women in physics has grown very slowly over the previous decades. Currently, about 20\% of the assistant professors in PhDgranting departments are women, and about $23 \%$ are women in bachelor'sgranting departments (Figure 5), which is higher than the percentage of women among recent PhD graduates. This is further evidence that there is no systematic bias against hiring women.

## Table 5

Probability that Faculty is All Women or All Men For Specific Departments Sizes and \% Women Overall

|  |  | Expected Proportion of Departments <br> with Faculty Members which ... |  |
| :---: | :---: | :---: | :---: |
| Department <br> Profile | Overall <br> Matches ... | Women |  |
|  | Are All the Same <br> Sex (\%) | Have Members of <br> Both Sexes (\%) |  |
|  | 16 | 49 | 51 |
| Granting | 23 | 37 | 63 |
| Departments | 50 | 20 | 80 |
| PhD- | 11 | 12 | 88 |
| Granting | 20 | 3 | 97 |
| Departments | 50 | $<1$ | $>99$ |

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This does not mean that selected individual departments do not discriminate or have environments that are less than welcoming to women.

Because of the large number of small departments, even if half of all faculty members were women, we would still expect to see over 100 bachelor's-granting physics departments have same-sex faculties.

Figure 5

The proportion of women at the assistant professor rank in bachelor'sgranting physics departments is actually higher than that of recent PhD graduates.

We must also recognize that having at least one woman among the faculty in more departments results in an increased number of women who are the only woman in their department.

Proportion of Professorial-Rank Faculty Who are Women Bachelor's- and PhD-Granting Physics Departments, 2009-10 Academic Year

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## Conclusions

The sex composition of a physics department is the result of a multitude of events, some recent and some that go back many years. Based on these simulation results, though, we should not accept the absence of women among professorial-rank faculty in a single department to be prima facie evidence of a bias against women.

Instead, we would do well to consider departments' and institutions' environments in evaluating the situation of women in physics. Even in physics departments with women faculty members, women could experience hostile climates, lack of resources to do research, and limited professional opportunities, all of which can have a cumulative negative effect on their careers.

We must also recognize that having at least one woman among the faculty in more departments results in an increased number of women who are the only woman in their department. Given the current representation of women among all physics faculty, this isolation of women faculty members is the, perhaps unintended, tradeoff that occurs when women, representing a small proportion of all physics faculty members, are found in more departments than expected. The only way to reduce the number of women faculty members who are the only woman in their department while continuing to have women in these departments is to increase the representation of women overall. However, even if women ultimately comprise half of all physics faculty members, we would still expect to find
some departments with no women among their faculty - and some with no men.

While counting the number of departments with no women is not a valid measure of gender equity, we do not mean to provide a convenient explanation for departments that have no gender variation. Instead, the issue of gender equity in physics is more complex and nuanced. It cannot be distilled into any single measure.

## About the Academic Workforce Survey

The Academic Workforce Survey of physics departments has been conducted every two years since 1986. Every four years, we collect data on the number of women in each department by their rank. For the data used in this paper, we contacted each of the 758 departments that awarded at least a bachelor's degree in physics by e-mail, mail, and certified mail. Follow-up contacts were made for departments that had not yet responded. Data collection began in March 2010 and ended in July 2010. We received responses from 707 departments ( $93 \%$ ). We offer our sincere gratitude to the responding departments. Without your help, we could not track these data or provide them to the community.


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